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Effects of thermophoresis particle deposition in free convection boundary layer from a horizontal flat plate embedded in a porous medium

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Abstract

The present contribution deals with the thermophoresis particle deposition effect on the free convection over a horizontal flat plate embedded in a fluid-saturated porous medium. In a certain sense, this study continues that by Chamka and Pop [A. Chamka, I. Pop, Effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium, Int. Commun. Heat Mass Transfer 31 (2004) 421–430], where the vertical flat plate configuration was analyzed. The governing equations are transformed into a set of coupled differential equations, which are solved numerically using a finite difference method. For various values of the problem parameters, graphs of the profile concentration in the boundary layer and of thermophoretic deposition velocity are presented.

1. Introduction

In the phenomenon of thermophoresis small sized (submicron) particles suspended in an isothermal gas acquire velocities relative to the gas in the direction of decreasing temperature. Thermophoresis is important when the particle sizes are small and the temperature gradients are large. When the wall is cold, the particles tend to deposit on the surface, while when the wall is hot the particles tend to repel from that surface.

In the field of viscous fluids there is a large body of papers dealing with the effect of thermophoresis particle deposition. We limit ourselves to a short description of the status-of-the art in the laminar regime. In the paper [1], by Epstein et al., thermophoretic deposition of particles from a vertical plate in free convection boundary layer flow was analyzed. Further, thermophoresis in the horizontal plate configuration was focused by Goren [2]. The thermophoretic transport of small particles in forced convection flow over inclined plates was studied by Garg and Jayaraj [3]. The paper by Jayaraj et al. [4] dealt with thermophoresis in natural convection with variable properties for a laminar flow of a viscous fluid over a cold vertical flat plate. Selim et al. [5] analyzed the effect of surface mass flux on mixed convection flow past a heated vertical flat permeable plate with thermophoresis, by considering a nonuniform surface mass flux through the surface. The combined effects of inertia, diffusion and thermophoresis on particle deposition from a stagnation point flow onto an axisymmetric wavy wafer have been examined by Wang [6]. In another very recent paper, [7], a theoretical model for the coupled transport mechanisms of diffusion, convection and thermophoresis was developed to describe the particle deposition onto a continuous moving wavy surface.

At present, to the author's best knowledge, two studies dealing with the thermophoresis effect in porous media were published: recently Chamka and Pop [8] looked to the effect of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium; the steady free convection over an isothermal vertical circular cylinder embedded in a fluid-saturated

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Nomenclature

C	concentration
D_m	mass diffusivity
f	dimensionless stream function
K	Darcy permeability
k_T	thermal diffusion ratio
Le	Lewis number, α_m/D_m
N	sustentation parameter
N_t	thermophoresis parameter
Ra_x	local Rayleigh number
u, v	Darcian velocities in the x and y-direction, respectively
v_T	thermophoretic deposition velocity
V_{tw}	thermophoretic deposition velocity at the wall
T	temperature
x, y	Cartesian coordinates normal to the plate and along it, respectively
α_m	thermal diffusivity
β_T	coefficient of thermal expansion
β_C	coefficient of concentration expansion
ϕ	dimensionless concentration
μ	dynamic viscosity
ν	kinematic viscosity
θ	dimensionless temperature
ρ	density
ψ	stream function

Subscripts

w	condition at wall
∞	condition at infinity

Superscript

$'$	differentiation with respect to η

porous medium in the presence of the thermophoresis particle deposition effect was analyzed in [9]. On the other hand, the impetuous research on convective flows in porous media is surveyed in the recents books by Nield and Bejan [10] and Ingham and Pop [11,12].

The objective of the present paper is to analyze the effect of thermophoresis particle deposition in free convection from a horizontal flat plate embedded in a porous medium.

2. Analysis

Consider the natural convection in a porous medium saturated with a Newtonian fluid bounded by a horizontal flat plate with constant wall temperature T_w and constant wall concentration C_w. The temperature and concentration of the ambient medium are T_∞ and C_∞, respectively. The x-coordinate is measured along the plate from its leading edge, and the y-coordinate normal to it. Several assumptions are used throughout the present paper: (a) the fluid and the porous medium are in local thermodynamic equilibrium; (b) the flow is laminar, steady-state and two-dimensional; (c) the porous medium is isotropic and homogeneous; (d) the properties of the fluid and porous medium are constants; (e) the Boussinesq approximation is valid and the boundary-layer approximation is applicable.

In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=-\frac{K}{\mu}\frac{\partial p}{\partial x},\quad v=-\frac{K}{\mu}\left(\frac{\partial p}{\partial y}+\rho g\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\frac{\partial(v_T C)}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

together with the Boussinesq approximation

$$\rho=\rho_\infty[1-\beta_T(T-T_\infty)-\beta_C(C-C_\infty)] \tag{5a}$$

where the thermophoretic deposition velocity in the y-direction is given by

$$v_T=-k\frac{\nu}{T}\frac{\partial T}{\partial y} \tag{5b}$$

and k is the thermophoretic coefficient. We remark that only the velocity component given by (5) is to be considered within the boundary-layer framework. The boundary conditions are

$$y=0: v=0, T=T_w, C=C_w \tag{6a}$$

$$y\to\infty: u\to 0, T\to T_\infty, C\to C_\infty \tag{6b}$$

Introducing the stream function ψ in the usual way, in order to identically satisfy the continuity equation, and using the dimensionless quantities

$$\psi=\alpha Ra_x^{1/3}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\quad \eta=Ra_x^{1/3}\frac{y}{x} \tag{7}$$

Eqs. (1)–(4) become

$$f''-\frac{2}{3}\eta\theta'-\frac{2}{3}N\eta\phi'=0 \tag{8}$$

$$\theta''+\frac{1}{3}f\theta'=0 \tag{9}$$

$$\frac{1}{Le}\phi''+\frac{1}{3}f\phi'+\frac{kPr}{N_t+\theta}\left(\theta'\phi'+\theta''\phi-\frac{\phi}{N_t}+\theta\theta'^2\right)=0 \tag{10}$$

where the sustentation parameter N, the thermophoresis parameter N_t, the local Rayleigh number Ra_x, the Prandtl number Pr and the Lewis number Le are defined as follows

$$N=\frac{\beta_C(C_w-C_\infty)}{\beta_T(T_w-T_\infty)},\quad N_t=\frac{T_\infty}{T_w-T_\infty},$$
$$Ra_x=\frac{\rho_\infty g\beta_T K(T_w-T_\infty)x}{\mu\alpha},\quad Pr=\frac{\nu}{\alpha_m},\quad Le=\alpha_m/D_m \tag{11}$$

Substituting θ'' from (9) into (10) leads to

$$\frac{1}{Le}\phi''+\frac{1}{3}f\phi'+\frac{kPr}{N_t+\theta}\left(\theta'\phi'-\frac{1}{3}f\phi\theta'-\frac{\phi}{N_t+\theta}\theta'^2\right)=0 \tag{12}$$

We remark to this end that, if $Le\neq 1$, Eq. (4) there must be proportional to $\partial^2C/\partial y^2$.

The set of ordinary differential equations (8), (9) and (12) must be solved along the following boundary conditions

$$f(0)=0,\quad \theta(0)=1,\quad \phi(0)=1 \tag{13a}$$
$$f'(\infty)=0,\quad \theta(\infty)=0,\quad \phi(\infty)=0 \tag{13b}$$

Of technical interest is the thermophoretic deposition velocity at the wall, which is given by the following expression

$$V_{tw}=-\frac{kPr}{1+N_t}\theta'(0) \tag{14}$$

3. Numerical analysis and results

Eqs. (8), (9) and (12) are numerically solved using a finite-difference technique which belongs to the Keller-box family. A nonuniform grid was adopted, which is concentrated towards the wall. The first step along the η-direction is 0.01, then the steps are increased in geometrical progression of ratio 1.01. After some trials we imposed a maximal value of η "at infinity" of 20. All the results presented in this paper are for $Pr = 0.72$ as in [8]. It follows that the remaining parameters of the problem are k, N, N_t and Le. During the trials of our numerical method, we obtained for $N = 0$ the values $f'(0) = 1.05576$ and $\theta'(0) = -0.42421$, while the value reported in [10], page 131, Table 5.2 (see also [13]) is $\theta'(0) = -0.420$, which are in good agreement.

Figs. 1 and 2 show the effects of N on concentration profiles for $k = 0.5$, $N_t = 100$, when $Le = 1$ and $Le = 10$, respectively. In comparison with the vertical case (Fig. 2 from [8]) the behaviour of the concentration profiles shown in our Fig. 2 is quite similar. On the other hand, by comparing Figs. 1 and 2 one can readily see that the concentration boundary layer is thicker for $Le = 1$ than for $Le = 10$.

The effects of Le and N on thermophoretic deposition velocity V_{tw} can be seen in Fig. 3 when $k = 0.5$ and $N_t = 100$. Once again, it is instructive to compare our results with those obtained by Chamka and Pop [8], see Fig. 1 from that paper, where the parameters have the same values as ours. The general behaviour is the same, but the values of V_{tw} are larger in our case.

In Fig. 4 there is represented the thermophoretic deposition velocity as a function of k and N when $Le = 1$ and $N_t = 100$. A similar plot is shown in Fig. 5 but now for $Le = 10$ and $N_t = 100$. As the Lewis number decreases difficulties in reaching the convergence of the numerical scheme are encountered for larger values of k ($k = 0.6$ for $Le = 1$ in Fig. 4). One can remark that the range of allowable N is extended as the Lewis number Le increases ($N_{max} = 15$ in Fig. 4 and $N_{max} = 100$ in Fig. 5). Otherwise, the thermophoretic deposition velocity increases as k increases, at a fixed value of N, as in the vertical case.

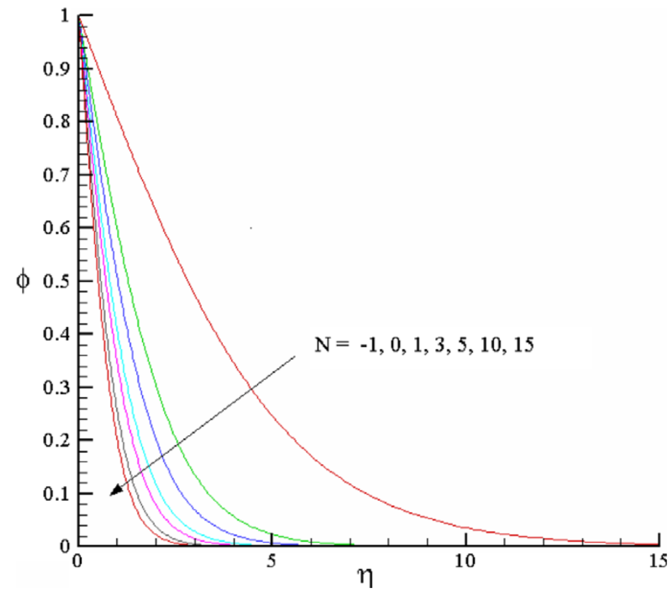

Fig. 1. Effects of N on concentration profiles, $Le = 1$, $k = 0.5$, $N_t = 100$.

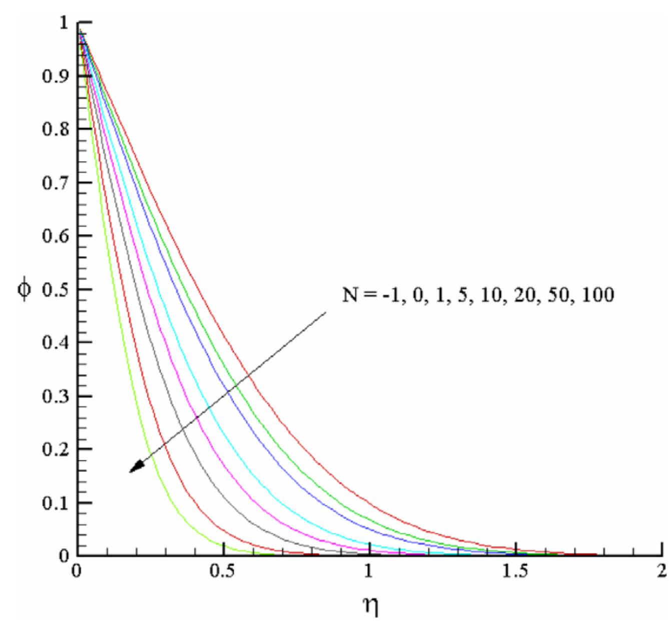

Fig. 2. Effects of N on concentration profiles, $Le = 10$, $k = 0.5$, $N_t = 100$.

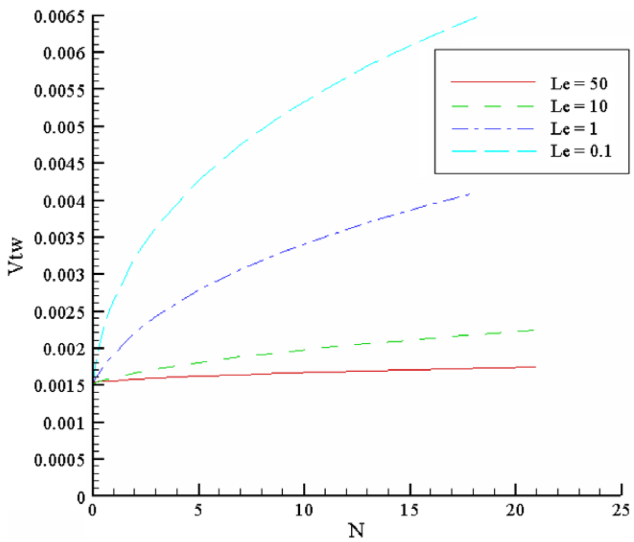

Fig. 3. Effects of *Le* and *N* on thermophoretic deposition velocity, $k = 0.5$, $N_t = 100$.

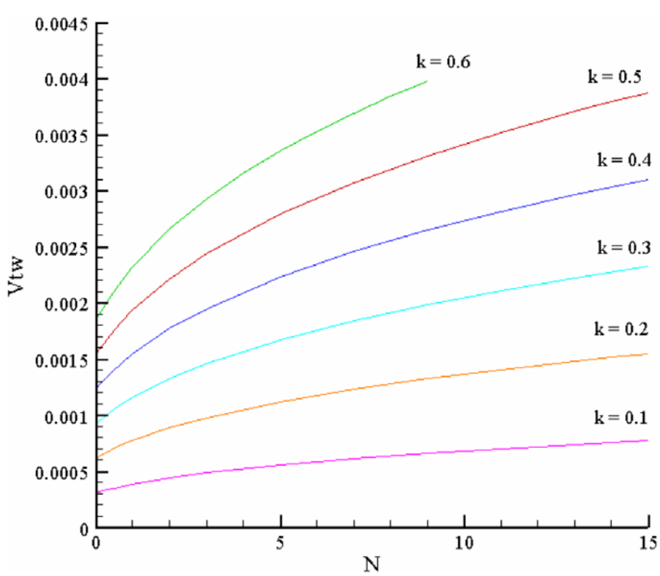

Fig. 4. Effects of k and N on thermophoretic deposition velocity, $Le = 1$, $N_t = 100$.

Finally, in Fig. 6 the effects of N_t and N on thermophoretic deposition velocity are depicted when $Le = 10$ and $k = 0.5$. Once again one can remark a similar behaviour with the vertical case, Fig. 5 from [8]: as N_t increases, the deposition velocity decreases, at a fixed value of N.

4. Final remarks

The effect of thermophoresis particle deposition on heat and mass transfer produced in steady, laminar boundary-layer flow past a horizontal flat plate was analyzed in this paper. There are five parameters involved in the final form of the mathematical model. The problem can be extended on many directions, but the first one seems to be to consider a power law variation of the wall temperature with

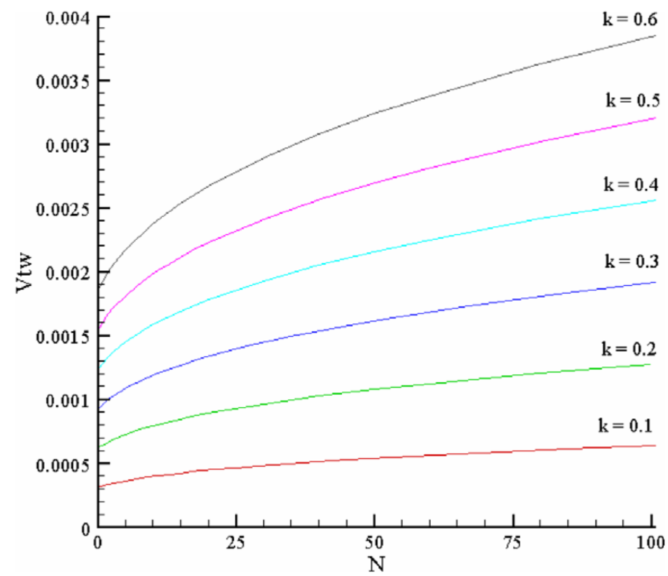

Fig. 5. Effects of k and N on thermophoretic deposition velocity, $Le = 10$, $N_t = 100$.

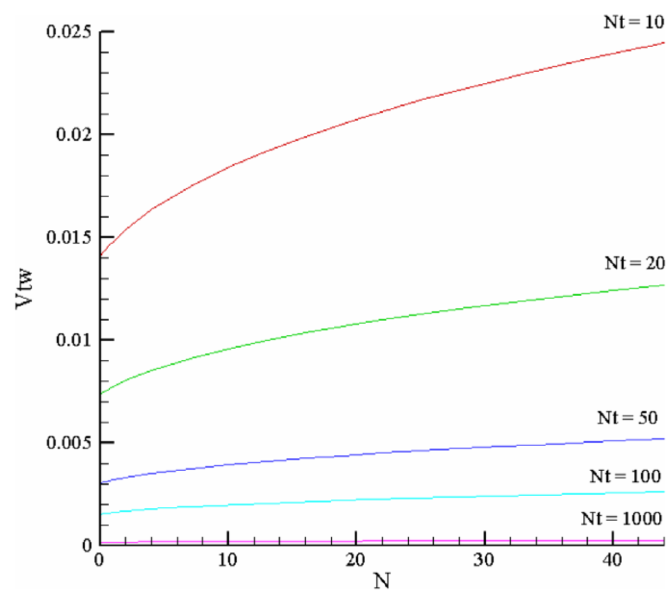

Fig. 6. Effects of N_t and N on thermophoretic deposition velocity, $Le = 10$, $k = 0.5$.

x: $T = T_w + Ax^{\lambda}$, where the "+" and "−" signs are for a heated plate facing upward and for a cooled plate facing downward, respectively, and A is a positive constant, see the classic paper [13]. However, due to the large number of parameters involved in the problem, we refrain ourselves to introduce here this new parameter, which is λ, although we made a number of runs for several values of this parameter (0.5, 1, 1.5 and 2.0, according to [13]). The general behaviour portrayed in the previous section remains, with specific problems of convergence, which limit in some cases the allowable range for N and k in obtaining numerical solutions of the problem.

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